

Spin control in semiconductor quantum wires

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We show that spin-flip rotation in a semiconductor quantum wire, caused by the Rashba and the Dresselhaus interactions (both of arbitrary strengths), can be suppressed by dint of an in-plane magnetic field. We found a new type of symmetry, which arises at a particular set of intensity and orientation of the magnetic field and explains this suppression. Based on our findings, we propose a transport experiment to measure the strengths of the Rashba and the Dresselhaus interactions.

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Spin-polarized transport in semiconductor nanostructures is the main topic in spintronics due to great interests to both basic research and device application [1, 2]. Spin-orbit interactions present in semiconductor structures provide a promising way to spin manipulation in bulk semiconductors [3], two-dimensional (2D) electron gases [4], and quantum dots [5]. However, these interactions cause decay of spin polarization [6], since the spin-orbit coupling breaks the total spin symmetry. The effect of spin relaxation produced by the interplay between the Dresselhaus [7] and Rashba [8] spin-orbit interactions (RDI) has been studied in a few publications (cf.[9, 10, 11, 12, 13, 14]). It was found by Schliemann *et al* [10] that at zero magnetic field in 2D semiconductor nanostructures for equal strengths of the RDI there is an additional symmetry [14]. As a consequence, the orbital motion is decoupled from the spin evolution. If this resonant condition is not active, the spin dynamics is influenced by the different spin relaxation mechanisms related to orbital scattering processes. In this paper we discuss another spin symmetry that arises at certain conditions at nonzero magnetic field in the plane and arbitrary strengths of the spin-orbit terms in a quantum wire. In virtue of this symmetry the spin-flip rotation is suppressed at arbitrary polarization of the injected electrons. By setting these conditions 'on' and 'off', the flow of a certain spin polarization through the device is either allowed or destroyed, thus, defining a transistor-like action for the spin.

We consider the conduction band of a 2D semiconductor quantum well within the effective mass approximation. The wire geometry is defined by a transversal potential $V(y)$: $\mathcal{H}_0 = (p_x^2 + p_y^2)/2m^* + V(y) = \mathcal{H}_y + p_x^2/2m^*$. The Dresselhaus interaction has, in general, a cubic dependence on the momentum of the carriers. For a narrow $[0, 0, 1]$ quantum well, it reduces to the 2D linear momentum dependent term $\mathcal{H}_D = \beta (p_x \sigma_x - p_y \sigma_y) / \hbar$ (β is the interaction strength). In the asymmetric quan-

tum wells the Bychkov-Rashba interaction has the form: $\mathcal{H}_R = \alpha (p_y \sigma_x - p_x \sigma_y) / \hbar$, where α is the corresponding strength. Our system Hamiltonian reads as

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_D + \mathcal{H}_R + \mathcal{H}_Z, \quad (1)$$

where we include the effect of the in-plane magnetic field by means of the Zeeman interaction $\mathcal{H}_Z = g^* \mu_B B (\cos \theta \sigma_x + \sin \theta \sigma_y) / 2 = \varepsilon_z (\cos \theta \sigma_x + \sin \theta \sigma_y) / 2$. Here, θ represents the in-plane orientation of the magnetic field with the intensity B , g^* is the effective gyromagnetic factor and μ_B is the Bohr's magneton. Note that none of the interactions break the translational invariance in the longitudinal coordinate. Therefore, the eigenstates are chosen to have a well-defined longitudinal momentum ' $\hbar k$ '

$$\Psi_{nks}(\vec{r}) = e^{ikx} \begin{pmatrix} \chi_{nks\uparrow}(y) \\ \chi_{nks\downarrow}(y) \end{pmatrix} =: e^{ikx} \chi_{nks}(y), \quad (2)$$

where n, k, s stand for transversal, longitudinal and spin quantum numbers. As a result, the Hamiltonian (1) is transformed to the effective one for the transversal coordinate for a given value of ' k '

$$\begin{aligned} \mathcal{H} = & \mathcal{H}_y + \frac{p_y}{\hbar} (\alpha \sigma_x - \beta \sigma_y) + \\ & + \left[\left(\beta k + \frac{\varepsilon_z}{2} \cos \theta \right) \sigma_x - \left(\alpha k - \frac{\varepsilon_z}{2} \sin \theta \right) \sigma_y \right]. \end{aligned} \quad (3)$$

Two different spin-dependent terms can be distinguished within this Hamiltonian; one is involving the transversal component of the momentum and the other contains the effective Zeeman-like term including contributions from the RDI. If both terms are parallel in the spin space, a symmetry arises and the spin is totally decoupled from the orbital motion. In order to set this symmetry, it is required to fulfill the following condition

$$\frac{2\alpha k_0}{\varepsilon_z} \left[1 - \left(\frac{\beta}{\alpha} \right)^2 \right] = \sin \theta + \frac{\beta}{\alpha} \cos \theta. \quad (4)$$

Once Eq.(4) is fulfilled, the spin operator $\mathcal{S}_{xy} = \alpha\sigma_x - \beta\sigma_y$ commutes with the resulting Hamiltonian

$$\mathcal{H} = \mathcal{H}_y + \left[\frac{p_y}{\hbar} + \frac{1}{\alpha}(\beta k_0 + \varepsilon_z \cos \theta/2) \right] (\alpha\sigma_x - \beta\sigma_y), \quad (5)$$

i.e., $[\mathcal{H}, \mathcal{S}_{xy}] = 0$. Consequently, the spin symmetry is set up for transversal eigenstates having longitudinal momentum $k = k_0$. According to Eq.(4), this can be done by tuning a proper intensity ($\sim \varepsilon_z$) and an orientation of the applied in-plane magnetic field for given strengths α and β . It is noteworthy that this property is valid for any transversal potential defining the wire geometry, since the symmetry arises from the relation between the RDI and the Zeeman interaction in conjunction with the longitudinal translational invariance. In addition, there is an extra degree of freedom, since the RDI strengths (for example, α) can be modified as well in order to fulfill the condition (4).

In virtue of the spin symmetry, the spinorial part of the eigenstates can be expressed as

$$\chi_s = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ s e^{-i\phi} \end{pmatrix}, \phi = \text{atan}(\beta/\alpha), \quad (6)$$

where thereafter $s = \pm 1$. These eigenspinors correspond to the in-plane orientation of the spin, where the particular orientation is determined by the ratio between the strengths of the both spin-orbit mechanisms. Note that in the Hamiltonian (5) the spin-dependent term, linear in the transversal momentum ' p_y ', can be eliminated by re-defining the origin of the transversal momentum for each spin state. The only effect of this term on the energy spectrum is a constant shift that may be neglected by changing the energy origin.

At the condition (4) hold fixed, the spectrum of the system is composed of that corresponding to the spin-independent orbital motion ' \mathcal{H}_0 ' (ε_{nk}^0), the constant shift and a contribution arising from a combination of the RDI strengths and the Zeeman interaction

$$\varepsilon_{nk_0s} = \varepsilon_{nk_0}^0 - \frac{m^*}{2\hbar^2} r^2 + \frac{s}{\alpha} \left(\beta k_0 + \frac{\varepsilon_z}{2} \cos \theta \right) r. \quad (7)$$

Here we introduced the absolute magnitude of the RDI strength vector $r = \sqrt{\alpha^2 + \beta^2}$. The above contribution represents a constant spin splitting for the eigenstates and its value depends on the longitudinal momentum, the RDI strengths, the particular orientation and the intensity of the applied magnetic field. At the preserved symmetry the eigenstates (2) take the form:

$$\Phi_{nk_0s}(\vec{r}) = e^{-isy/l_{RDI}} e^{ik_0x} \psi_n^0(y) \chi_s, \quad (8)$$

where $\psi_n^0(y)$ are the eigenstates of \mathcal{H}_y . We have also defined the length $l_{RDI} = \hbar^2/(m^*r)$ giving the characteristic scale for the RDI strengths.

At given Fermi energy E_F eigenstates (2) have a few real longitudinal momentum k . Some of them have $k > 0$

(propagating right), while the others have $k < 0$ (propagating left). One of those k could satisfy the condition (4) by the adjusted magnetic field and, consequently, the corresponding spinor does not depend on coordinates. However, even a small mixing between the selected state and those that propagate in the same direction but have different k leads to the spin precession in the process of the propagation. To suppress unwanted k -values one can adjust the Fermi energy (or the potential $V(y)$) to have only four real longitudinal momenta. Next, the tuning of the intensity and the orientation of the magnetic field enables us to have two eigenstates (with $s = \pm$ in Eq.(7)), propagating in the same direction, with the same energy and k_0 . From Eqs.(4), (7) one obtains that such a possibility can be realized, if the components of the magnetic field are proportional to the components of the RDI vector:

$$\frac{\varepsilon_z}{2} \cos \theta_0 = -k_0\beta, \quad \frac{\varepsilon_z}{2} \sin \theta_0 = k_0\alpha. \quad (9)$$

As a result, there is no a spin-flip process for any superposition of these eigenstates. Note that, in contrast to the spin-field transistor proposed in Ref.[10] whose effect is based on a particular input spin polarization, in our case the spin-flip is absent for an arbitrary input spin polarization. Also, the magnetic field leads to a nonequivalence of the electron transport from the left to the right and vice versa: Eq. (9) is fulfilled for $-k_0$ at the condition $\theta_0 \rightarrow \theta_0 + \pi$.

To illuminate the found effect in the electron transport we perform numerical calculations of the S -matrix in the tight-binding model (cf [15]). To proceed we use a square lattice $n = n_x\hat{x} + n_y\hat{y}$ (\hat{x} and \hat{y} are vectors of a length a_0 in x and y directions, respectively; a_0 is the lattice constant, n_x and n_y are integers). Within this approach our Hamiltonian (1) has the following form

$$\begin{aligned} \mathcal{H}_0 &= \sum_{n,\sigma} \epsilon_{n\sigma} c_{n\sigma}^\dagger c_{n\sigma} - \sum_{\langle nm \rangle, \sigma} t c_{n\sigma}^\dagger c_{m\sigma} \\ \mathcal{H}_D &= -\frac{\beta}{2a_0} \sum_n \left\{ i \left(c_{n\uparrow}^\dagger c_{n+\hat{x}\downarrow} + c_{n\downarrow}^\dagger c_{n+\hat{x}\uparrow} \right) - \right. \\ &\quad \left. \left(c_{n\uparrow}^\dagger c_{n+\hat{y}\downarrow} - c_{n\downarrow}^\dagger c_{n+\hat{y}\uparrow} \right) \right\} + \text{H.c.} \\ \mathcal{H}_R &= -\frac{\alpha}{2a_0} \sum_n \left\{ i \left(c_{n\uparrow}^\dagger c_{n+\hat{y}\downarrow} + c_{n\downarrow}^\dagger c_{n+\hat{y}\uparrow} \right) - \right. \\ &\quad \left. \left(c_{n\uparrow}^\dagger c_{n+\hat{x}\downarrow} - c_{n\downarrow}^\dagger c_{n+\hat{x}\uparrow} \right) \right\} + \text{H.c.} \\ \mathcal{H}_Z &= \frac{\varepsilon_z}{2} \sum_n c_{n\uparrow}^\dagger c_{n\downarrow} (\cos \theta - i \sin \theta) + \text{H.c..} \end{aligned} \quad (10)$$

Here $c_{n\sigma}^\dagger$ creates an electron at site n with spin σ and energy $\epsilon_{n\sigma} = 4t - V(n_y a_0)$, $t = \hbar^2/2m^*a_0^2$, and $\langle nm \rangle$ stands for nearest neighbors sites n and m . For the sake of illustration, we choose for the wire potential a hard wall one: $V(y) = 0$ for $0 < y < W$ and $V(y) = \infty$ otherwise.

Let us consider the system with a geometry shown in Fig.1. It consists of a finite scattering area with

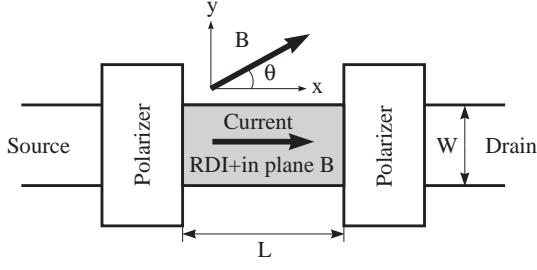


FIG. 1: Sketch of the 2D wire device.

two lateral contacts. Each contact is a narrow stripe with the width $W = 20a_0$ and, for simplicity, no spin-orbit couplings and no the magnetic field. The contacts are gated to have two active channels (spin up and down) with a conductance e^2/h in each. Thus, the RDI and the in-plane magnetic field present only in the scattering area of the length L and the width W . The experiment may consist of injecting a current I through the left contact (source) to the wire and measuring the voltage drop V_R generated in the right contact (drain). According to the Landauer-Buttiker formalism for linear response (cf [16]), the ratio V_R/I can be expressed by dint of the S -matrix elements $S_{m\sigma_2 n\sigma_1}$, where $n\sigma_1(m\sigma_2)$ denote the channels in the source (the drain). In our approach the spin resolved conductance between the source and the drain is determined as $G_{\sigma_1\sigma_2} = e^2/h \int dE [-f'(E - E_F)] \sum_{nm} |S_{m\sigma_2 n\sigma_1}|^2$, where $f = 1/[1 + \exp((E - E_F)/k_B T)]$. The conductance is calculated with the energy dependent S -matrix by direct solving the Schrödinger equation in a discretized space according to the method suggested in Ref.[17].

Note that the interfaces (the polarizers) between areas with and without the RDI introduce some uncontrollable excitations of all modes inside the scattering area. In particular, these excitations produce a superposition (with coefficients a_1 and a_2) of two eigenfunctions $\chi_{1,2}(y) \exp(ik_{1,2}x)$ with different longitudinal momenta and, therefore, rotate the spin during a transport along the x -axis. Indeed, one has the following expectation values

$$\langle S_x \rangle = \text{Re}\xi, \quad \langle S_y \rangle = \text{Im}\xi, \quad (11)$$

where $\xi = (a_1^* \chi_{1\uparrow}^* + a_2^* \chi_{2\uparrow}^*)(a_1 \chi_{1\downarrow} + a_2 \chi_{2\downarrow}) \times \exp(i(k_2 - k_1)x)$. Evidently, for equal longitudinal momenta, the expectation values are independent on the x coordinate. The results (see Fig.2, top) manifest a single common minimum of the spin-flip conductance ($\sim 10^{-3}e^2/h$) for one value of the magnetic field orientation but for different sample lengths, at a given intensity of the magnetic field and at zero temperature. At this value Eq.(9) holds, indeed. For another angles there is the mixing of wavefunctions with different k which leads to the electron spin rotation in the sample. Fig.3 illuminates the dependence of longitudinal momenta $k(\theta)$ on the magnetic field orientation. At a particular value of the angle

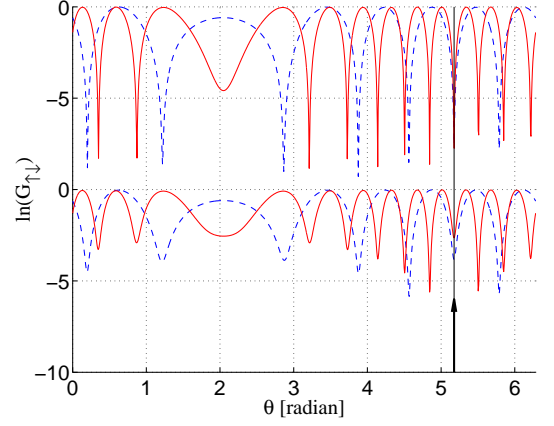


FIG. 2: (Colour online) The logarithm of the spin-flip conductance (in units of e^2/h) as a function of the magnetic field orientation θ (the intensity $B = 1.8\text{T}$) for different sample lengths: $L=31W$ (solid line), $L=17W$ (dashed line). Temperature is $T=0\text{K}$ (top) and $T=1\text{K}$ (bottom). The incoming electrons are polarized along z -axis. The set of parameters are typical for InAs: $E_F=10\text{ meV}$, $W=45\text{ nm}$, $\alpha=20\text{ meV nm}$, $\beta=10\text{ meV nm}$, $g^*=-14.9$, $m^*=0.023m_e$. The arrow indicates the unique position of the angle θ_0 for non-spin-flip conductance, independent on the sample length.

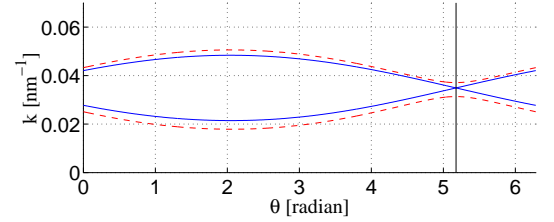


FIG. 3: (Color online) The longitudinal momentum k as a function of the orientation of the magnetic field (the angle θ) which intensity is subject to Eq.(9) (solid line) and is slightly different (dashed line). At the value $\theta_0 = 5.176$ (vertical line) the both positive k coincide. The parameters are the same as for Fig.2.

θ_0 two wavenumbers coincide. However, the change of the magnetic field intensity (the value of ε_z) leads to avoided crossing of two curves $k(\theta)$.

In real experiments the injected beam consists of electrons with different energies due to, for example, a nonzero temperature. The temperature induces a small mixture of spin-flip components and results in the increase of the spin-flip conductance (see Fig.2, bottom). However, it does not affect the angle value at which the minima occur simultaneously in the two samples at temperature $T = 1\text{ K}$.

Measurements of the RDI strengths is a subject of intensive experimental efforts [4, 18]. For example, the ratio α/β could determined with the aid of the analysis of photocurrents [18]. Note that Eq.(9) enables us to

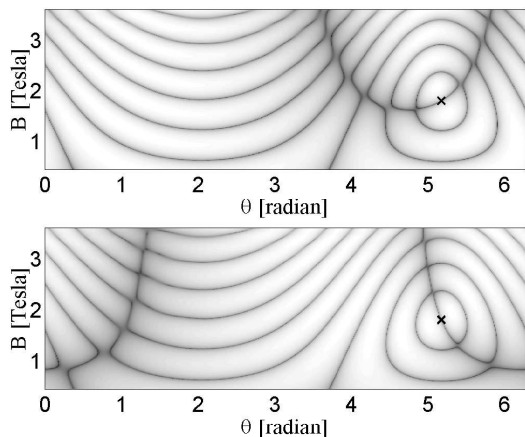


FIG. 4: The logarithm of the spin-flip conductance (in units of e^2/h) as a function of the intensity B and orientation θ of the magnetic field for the sample length $L = 31W$ and temperature $T=0K$ (the darker is line the lesser is the conductance). The input polarization is along x axis (top) and y axis (bottom). The minimum due to Eq.(9) is pointed by \times mark. The parameters are the same as for Fig.2.

determine the strengths of the Rashba and Dresselhaus interactions. We propose to use a wire with a length determined by the condition $kL/(2\pi) \sim 5$. According to our analysis (see Fig.4), such a system produces a few, well resolved spin-flip conductance minima. This condition helps also to diminish the effect of evanescent modes. The Fermi energy and the transversal potential $V(y)$ should be taken to support only four propagating modes (two with a positive k , going to the right and two with a negative k , going to the left). The measure of the spin-flip conductance provides a set of spin-flip minima at

different intensities and different orientations of the magnetic field, at a fixed input polarization. Taking another polarization and repeating the same measurement, one obtains a different pattern for the location of the minima. As an example, we calculate the spin-flip conductance for two input polarizations – along x - and y - axis (see Fig.4). One obtains the required minimum which is subject to Eq.(9) at the same angle and the same intensity in different setups, since the effect is independent on the polarization. One might repeat measurements for different sample lengths, since the minimum position is independent on the length too. To diminish the effect of multiple reflection from the polarizers we suggest to use the same polarization direction in the both polarizers.

In conclusion, we found the condition (Eq.(4)) to decouple the spin and the orbital motion of electrons in a quantum wire with the in-plane magnetic field and arbitrary Rashba and Dresselhaus strengths. At this condition there is the spin symmetry in an arbitrary transversal potential defining the wire geometry. Furthermore, at the specific condition (9) the magnetic field cancels the RDI for the electron momentum k_0 . As a result, during the electron transport through the wire the spin-flip rotation is absent for any chosen polarization. We propose the experiment to measure the strengths of the Rashba and the Dresselhaus interaction by finding the minimum of the spin-flip conductance, which should occur at the condition (9).

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- [1] I. Zutic, J. Fabian, and S. Das Sarma, Rev. Mod. Phys. **76**, 323 (2004).
 - [2] D. D. Awschalom and M. E. Flatté, Nature Phys. **3**, 153 (2007).
 - [3] Y. Kato *et al.*, Nature (London) **427**, 50 (2004).
 - [4] L. Meier *et al.*, Nature Phys. **3**, 650 (2007).
 - [5] K. C. Nowack *et al.*, Science **318**, 1430 (2007).
 - [6] M. I. D'yakonov *et al.*, JETP **63**, 655 (1986).
 - [7] G. Dresselhaus, Phys. Rev. **100**, 580 (1955).
 - [8] Y. A. Bychkov and E. I. Rashba, J. Phys. C **17**, 6039 (1984).
 - [9] N. S. Averkiev, L. E. Golub, and M. Willander, J. Phys.: Condens. Matter **14**, R271 (2002).
 - [10] J. Schliemann, J. C. Egues, and D. Loss, Phys. Rev. Lett. **90**, 146801 (2003).
 - [11] J. Kainz, U. Rössler, and R. Winkler, Phys. Rev. B **68**, 075322 (2003).
 - [12] M. Duckheim and D. Loss, Phys. Rev. B **75**, 201305 (R) (2007).
 - [13] M. Ohno and K. Yoh, Phys. Rev. B **77**, 045323 (2008).
 - [14] B. A. Bernevig, J. Orenstein, and S.-C. Zhang, Phys. Rev. Lett. **97**, 236601 (2006).
 - [15] D. K. Ferry and S. M. Goodnick, *Transport in Nanostructures* (Cambridge University Press, New York, 1997).
 - [16] Y. Alhassid, Rev. Mod. Phys. **72**, 895 (2000).
 - [17] T. Ando, Phys. Rev. B **44**, 8017 (1991).
 - [18] S. D. Ganichev *et al.*, Phys. Rev. Lett. **92**, 256601 (2004).